FALL 2006 — MA 227-8B — TEST 1

Name: _____

1. Part I

There are 4 problems in Part 1, each worth 4 points. Place your answer on the line to the right of the question. No partial credit will be given on Part 1 problems, only your answer on the answer line will be graded.

(1) Find the dot product of the vectors $\langle 3, -1, 4\rangle$ and $\langle 2, 3, -1\rangle$.

Answer: -1.

(2) Find the cross product of the vectors $\langle 0, 0, 2\rangle$ and $\langle 3, 0, 5\rangle$.

Answer: $6\,\mathbf{j}$.

(3) Find the derivative of the vector function $\langle \cos t, \sin(5t), t\rangle$.

Answer: $\langle -\sin t, 5\cos 5t, 1\rangle$.

(4) Find the indefinite integral $\int (2\,\mathbf{i} - 3t^2\,\mathbf{k})\,dt$.

Answer: $2t\,\mathbf{i} - t^3\,\mathbf{k} + C$.

2. Part II

There are 3 problems in Part 2, each worth 8 points. On Part 2 problems partial credit is awarded where appropriate. Your solution must include enough detail to justify any conclusions you reach in answering the question.

(1) Find the curvature of the graph of the function $y = e^{2x}$ at the point $(0, 1)$.

Answer: $4/5^{3/2}$.

(2) The position vector of a point moving is space is given by $\mathbf{r}(t) = 2t\,\mathbf{i} - \sin t\,\mathbf{j} - \cos t\,\mathbf{k}$. Find the unit tangent vector $\mathbf{T}$ and the unit normal vector $\mathbf{N}$ at time $t = 0$. Compute the scalar product of $\mathbf{T}$ and $\mathbf{N}$. Interpret the result geometrically.

Answer: $\mathbf{T}(0) = \langle 2/\sqrt{5}, -1/\sqrt{5}, 0\rangle$ and $\mathbf{N}(0) = \langle 0, 0, 1\rangle$; their dot product is zero, which means they are orthogonal.

(3) A ball is thrown at an angle of 30° to the ground. If the ball lands 50 m away, what was the initial speed of the ball? Hint: $\sin 30^\circ = 1/2$. You do not need to compute the numerical value of the initial speed v, it is enough to obtain the answer in terms of radicals. For example if you get $v_0 = \sqrt{12g/\sqrt{5}}$, it will be fine.

Answer: $v_0 = \sqrt{100g/\sqrt{3}}$.