

Name: \_\_\_\_\_

Signature: \_\_\_\_\_

**You must show your work and give reasons for your answers!**

**Good luck.**

**Part I.** All problems in Part I are worth 6 points.

Evaluate the following integrals:

(1)  $\int x^2 \sin(x^3 + 1) dx$

(2)  $\int x \cos(x) dx$

(3)  $\int \frac{1}{x^2-1} dx$

(4) **Set up** an integral for the volume of revolution obtained by rotating the area bounded by the graphs of  $y = \sin(x)$ ,  $y = 1$ ,  $x = 0$  and  $x = \pi/2$  around the line  $y = 3$ .

(5) Find the interval and radius of convergence for the series  $\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$

(6) Find the sum of the series  $1 - x^2 + x^4 - x^6 + x^8 \dots$ . For which  $x$  is this valid?

(7) Find the angle between the vectors  $\langle 1, 0, 0 \rangle$  and  $\langle 1, 1, 1 \rangle$ .

(8) Are the vectors  $\langle 1, 1, 1 \rangle$ ,  $\langle 1, 0, 0 \rangle$  and  $\langle 0, 1, 0 \rangle$  coplanar?

(9) Find the equation of the plane through the point  $(-1, 0, 1)$  and perpendicular to the

$$\text{line } \begin{cases} x = 1 - 2t \\ y = 1 + t \\ z = 1 - 3t \end{cases}$$

(10) Use a Riemann sum with  $n = 4$  terms, and the midpoint rule, to estimate the value of  $\int_0^{1/10} \frac{1}{1+x^5} dx$ . [You don't need to add all the numbers in the sum.]

**Part II** All problems in Part II are worth 10 points

- (11) Use series to estimate the value of  $\int_0^{1/10} \frac{1}{1+x^5} dx$  with an error less than  $10^{-7}$ .
- (12) The intersection of solid  $S$  with planes perpendicular to the  $x$ -axis are round disks, whose diameters are contained in the  $xy$ -plane and stretch from the graph of  $y = x$  to  $y = x^2$  for  $0 \leq x \leq 1$ . Find the volume of  $S$ .

- (13) Suppose an upside down pyramid has a square base of length 2m and a height of 3m, and is full of water. Find the work done in pumping all of the water over the upper edge. [You can use that water has a density of  $1000 \text{ kg/m}^3$  and that the gravitational acceleration  $g \approx 10 \text{ m/sec}^2$ .]

- (14) Find the distance between the lines  $\begin{cases} x = 1 \\ y = 1 + t \\ z = 2 \end{cases}$  and  $\begin{cases} x = s \\ y = 2 \\ z = 3 \end{cases}$