# MA 227: Calculus III
## Final Test, December, 2005

Time allotted: 150 min.

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1. The solid $E$ in space lies above the surface $z = (x^2 + y^2)$ and below the sphere $x^2 + y^2 + z^2 = 6$. Calculate its volume.

10 points

2. Evaluate

$$\iiint_E z dV,$$

where $E$ lies between the sphere $x^2 + y^2 + z^2 = 4$ and the cylinder $x^2 + y^2 = 1$ in the first octant.

10 points

3. The lamina $D$ is defined by the inequalities $0 \leq x \leq 1$, $0 \leq y \leq 1$, and its mass density function is given by $\rho(x,\, y) = x + y$. Compute its mass and the center of mass.

10 points

4. The solid $B$ lies inside the cylinder $x^2+y^2 = 1$ and inside the sphere $x^2+y^2+z^2 = 9$. Calculate its volume.

10 points

5. Evaluate the integral by reversing the order of integration.

$$\int_0^4 \int_{y^{1/2}}^2 e^{x^3}\, dx dy.$$

10 points

6. Calculate the integral

$$\int_1^2 \int_1^2 (\frac{x^2}{y} + \frac{y^2}{x}) dy dx.$$

10 points

7. Find the minimum and maximum values of the function $f(x, y, z) = yz + xy$ subject to the constraints $xy = 3$ and $y^2 + z^2 = 1$.

10 points

8. We know that $x$, $y$, and $z$ are positive numbers the sum of which is equal to 1. Maximize the value of $xy^2z^2$.

10 points

9. Find the points on the ellipsoid $x^2 + y^2 + 9z^2 = 41$ where the normal line is parallel to the line connecting the points $(3, -1, 0)$ and $(5, -2, 18)$.

10 points

10. Let $z = y^2 \tan x$, $x = t^2 uv$, $y = u + tv^2$. Find $\partial z/\partial t$, $\partial z/\partial u$, and $\partial z/\partial v$ when $t = 2$, $u = 1$, $v = 0$.

10 points