MA 125 Fall 2005 Final Examination

Part One

There are ten Part One problems, worth 4 points each. Place your answer on the line to the right of the question. Space is provided between problems for you to work each problem, but no partial credit will be given on Part One problems, and only your entry on the answer line will be graded.

1. Determine
$$\lim_{x\to\infty} \frac{2x^3 - x^2 + 1}{5x^3 + x + 9}.$$
2. Determine
$$\lim_{x\to 0} \frac{1 - \cos(x)}{x^2}.$$
3. Let $f(x) = x\cos(x)$. Determine $f'(x)$.
4. Let $f(x) = e^{\sin(x)}$. Find $f'(x)$.
5. Let
$$g(x) = \frac{\sqrt{x}}{1+x}.$$
Find $g'(x)$.
6. Let $f(x) = (\ln(2x))^4$. Find $f'(x)$.
7. Let $f(x) = e^x g(x)$. Find $f'(0)$ if $g(0) = 2$ and $g'(0) = 5$.
8. Let $g(x) = x^4 - 24x^2 + 6x - 8$. Find all open intervals on which the graph of g is concave down.
9. Evaluate $\int_0^{\pi/4} \cos(x)dx$.
10. Evaluate $\int (1 + 4x^2)dx$.

Part Two

There are six Part Two questions, each worth ten points. A page is provided for you to work each problem. Attach additional pages if they contain material you judge to be an important part of your solution. Your solution must include enough detail to justify any conclusions you reach in answering the questionh. Partial credit may be awarded on Part Two problems where it is warranted.

1. (a) Find y' if $y(x)$ is defined implicitly by the equation $x^2 - y^2 + 2xy - 2x - 4y + 9 = 0$.

(b) Find the equation of the tangent to the graph of this equation at $(2, 3)$.

(c) Find the $x-$ coordinate of each point on the graph of this equation at which the tangent is horizontal, or show that there is no such point.

2. Calculate the area bounded by the $x-$ axis and the graph of $y = x^3 - x$ for $-1 \le x \le 2$.

3. Let $f(x) = 3x^5 - 5x^3$.

(a) Determine each open interval on which $f(x)$ is increasing, and each open interval on which $f(x)$ is decreasing.

(b) Determine each open interval on which the graph of f is concave up, and each open interval on which the graph of f is concave down.

(c) Find all local maxima and minima of $f(x)$.

(d) Sketch a graph of $y = 3x^5 - 5x^3$.

4. Two objects are connected in parallel in an electrical circuit. One object has resistance $R_1(t)$ at time t, and the other has resistance $R_2(t)$. The total resistance $R(t)$ in the circuit satisfies the relationship

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}.$$

(a) If R_1 increases at a rate of 1 ohm per second, and R_2 increases at 2 ohms per second, how fast is the total resistance changing when $R_1 = 40$ ohms and $R_2 = 80$ ohms?

(b) Is R increasing or decreasing at this instant?

5. A rectangular box with a square base and an open top is to have a volume of 32 cubic centimeters. Find the dimensions of the box that minimize the amount of material used.

6. A ball is propelled from ground level straight up into the air with an initial velocity of 40 feet per second. It is known that its height after t seconds is given by $h(t) = 40t - 16t^2$ feet.

(a) Determine the velocity $v(t)$.

(b) Determine the acceleration $a(t)$.

(c) What is the maximum height the ball will reach?

(d) Assuming that the ball was thrown at time $t = 0$, how many seconds will elapse before the ball returns to the ground?