FINAL EXAM
MA227
DECEMBER 2004

Name:

Closed Book. No calculators. Show your work.

1. (10 pts. each) Evaluate the following:
(a) If $z = e^y \sin x$ and

$$\left.\frac{dx}{dt}\right|_{t=0} = 2, \qquad x(0) = 0, \qquad \left.\frac{dy}{dt}\right|_{t=0} = 1 = y(0),$$

find

$$\left.\frac{dz}{dt}\right|_{t=0}.$$

Date: February 1, 2005.

(b) Calculate div$\mathbf{F}$ for

$$\mathbf{F} = \sin(x^2 + y)\ \mathbf{i} + ye^z\ \mathbf{j} + x \ln y\ \mathbf{k}$$

(c) Compute the curl $\mathbf{F}$ for $\mathbf{F} := (xe^y - z)\ \mathbf{j} + xyz\ \mathbf{k}$.

(d) Let E be the region described by $1 < x^2 + y^2 + z^2 < 4$. Evaluate the integral

$$\iiint_E z \, dV$$

by changing to spherical coordinates.

(e) Determine whether or not the vector field $\mathbf{F}(x, y, z) = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ is conservative. If it is conservative, find $f(x, y, z)$ in order that $\mathbf{F} = \nabla f$.

2. Find all local maxima, minima, and saddle points for

$$f(x, y) = y^3 - 6xy + x^3.$$

3. Find the volume of the solid bounded above by the surface $z = xy^2$ and below by the triangle in the xy-plane with vertices $(1, 0)$, $(0, 2)$, and $(2, 0)$.

4. Find the surface area of the part of the paraboloid $z = x^2 + y^2$ that lies between the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 9$.

5. Suppose that C consists of the line segment from $(0,0)$ to $(1,0)$, the line segment from $(1,0)$ to $(1,1)$, and the arc of the curve $x = y^2$ from $(1,1)$ to $(0,0)$. Use Green's Theorem to evaluate

$$\oint_C \left(xe^{3x} - 4y^2\right)\, dx + \left(2xy + y\sin y^2\right)\, dy.$$

Extra Credit: Let the surface S_1 be the part of the sphere $x^2 + y^2 + z^2 = 5$ that lies inside the cylinder $x^2 + y^2 = 1$ and above the xy-plane. Let S_2 be the part of the plane $z = 2$ that lies inside the cylinder $x^2 + y^2 = 1$. If for some vector field $\mathbf{F}$,

$$\iint_{S_1} \text{curl } \mathbf{F} \cdot d\mathbf{S} = 3,$$

how does this fact relate to

$$\iint_{S_2} \text{curl } \mathbf{F} \cdot d\mathbf{S} = ?$$

(The amount of extra credit - if any - will depend upon how well you justify your answer.)

Test 1
MA227
September 2004
Name:

Closed Book. No calculators.
CIRCLE YOUR ANSWER. You must show your work and justify your answer to receive credit.

1. (a) (10 pts.) If $\mathbf{u} = \frac{1}{2}i + \frac{\sqrt{3}}{2}j$ find the directional derivative
$$D_u \sin(xy).$$

(b) (5 pts.) If $\mathbf{u} = a\mathbf{i} + b\mathbf{j}$, what are values of a and b (with $a^2 + b^2 = 1$) that maximizes $D_u \sin(xy)$ at the point with $x = 1$, $y = 0$?

2. (20 pts.) Let

$$f(x,y) = xye^{-(3x+2y)}.$$

Find **all** critical points and classify as local maxima, local minima, or saddle points.

3. A helix is described by

$$\mathbf{r}(t) := 3(\sin 2t)\mathbf{i} + 3(\cos 2t)\mathbf{j} - 4t\mathbf{k}.$$

(a) (6 pts.) Find the unit tangent vector $\mathbf{T}$ at the point $(0, 3, 0)$ on the helix.

(b) (6 pts.) Find the (principal unit) normal vector $\mathbf{N}$ at the point $(0, 3, 0)$ on the helix.

(c) (8 pts.) Find the plane containing the point $(0, 3, 0)$ and determined by the vectors $\mathbf{T}$ and $\mathbf{N}$ from parts (a) and (b), i.e., the *osculating plane*.

4. (15 pts.) Calculate the limit exists if it exists. If it does not exists, justify your answer.

   (a) (Hint: Change to polar coordinates.)

$$\lim_{(x,y)\to(0,0)} \frac{x^3 - xy^2}{x^2 + y^2}$$

   (b)

$$\lim_{(x,y)\to(0,0)} \frac{x^2 - 2xy}{4x^2 + y^2}$$

5. (15 pts.) Find $\frac{\partial f}{\partial t}$ if

$$f(x,y) = (\sin y)\ln(x^2+2), \quad \text{and} \quad x = 2\cos(st), \quad y = 3s - 2t.$$

6. An athlete puts a shot which leaves his hand 6 ft. above the ground at a 45 degree angle to the horizon and at a speed of $29\sqrt{2}$ ft./sec.
(a) (8 pts.) Find the position vector $\mathbf{r}(t)$, which describes the motion of the shot for any time t. [1]

(b) (4 pts.) How many seconds later does the shot hit the ground?

[1] Assume that the only force acting on the body is gravity. Hint: The acceleration of gravity is -32ft./sec.^{2}.

(c) (3 pts.) How far (horizontally) does the shot go?