MA 126, Calculus 2, UAB, Fall 2004.

FINAL EXAM

Duration 2 1/2 hours, Max. Points: 36.

For full credit in any of the nine problems: (1) justify your results, (2) be sure to address all parts of the given problem, and (3) frame or underline your final results. Write on these sheets or use extra paper if needed. Each problem is worth 4 points. Good luck!

1. Find the sum of the series.

(a) $2+\frac{1}{2}+\frac{1}{8}+\frac{1}{32}+\dots$

(b) $\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}$

2. Find the radius and the interval of convergence. Be sure to check the series for convergence at the *endpoints* of the interval!

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}(x-3)^n$

$$\sum_{n=1}^{\infty}(-1)^n\frac{1}{\sqrt{n}3^n}(x+1)^n$$

3. Find the Maclaurin series of the function $f(x)$ and its interval of convergence.

(a) $f(x) = \dfrac{x}{1 - x^2}$

(b) $f(x) = \ln(1 + x)$

4. (a) Use Taylor's inequality to find the maximum error possible in using the approximation

$$\cos x \simeq 1 - \frac{x^2}{2} \qquad \text{for } -1/2 \leq x \leq 1/2.$$

Why is $1 - x^2/2$ a reasonable approximation for $\cos x$ near $x = 0$ in the first place?
(b) Make an accurate sketch of both, the function $\cos x$ and the approximation $1 - x^2/2$ in the same xy-frame.

5. Find the 3rd degree Taylor polynomial of $f(x) = \sin x$ at $a = \pi/2$.

6. The picture shows the cap of a sphere with radius $r = 2$ and height $h = 1$. Find its volume.

7. (a) Compute the vector product of $\mathbf{a} = \langle -1, 3, 2 \rangle$ and $\mathbf{b} = \langle 3, 1, 9 \rangle$.
(b) Compute the area of the parallelogram spanned by these two vectors.

8. Find the angle between the z-axis and the plane $2x + y + 3z = 15$. First make a sketch of a plane and a vertical line (the z-axis) showing the angle of intersection. Label all parts of this sketch.

9. (a) Write the length of the curve

$$x(t) = e^t - t, \quad y(t) = 4e^{t/2}, \qquad \text{with } -8 \leq t \leq 3$$

as an integral. Simplify your result as much as possible.
(b) Evaluate the integral you found in part (a).