MA 126, Calculus 2 UAB, Fall 2004

TEST 1

Duration 70min;

Make sure to show all your work and underline the final results of each problem. Write your name on this sheet and use it as a cover page when you turn in your work. Do not write your results on this paper. Good luck!

1. The graph of f consists of two straight lines and a semi circle. Use it to evaluate each integral.

(a) $\int_3^4 f(x)\,dx$, (b) $\int_0^2 f(x)\,dx$, (c) $\int_1^4 f(x)\,dx$

2. (a) Use the properties of integrals to verify that

$$\int_1^3 \frac{1}{\ln(x)+2}\,dx \le 1$$

(b) Derive a good lower bound for integral in a similar way as the upper bound is derived. (E.g. -1 is a correct lower bound but not good enough.)

3. Write out the form of the partial fraction expansion of the function. Do not determine the numerical values of the coefficients.

(a) $\dfrac{4x-1}{(x+1)^2(x-3)}$ (b) $\dfrac{1+5x-x^2}{(x^2+2x+6)(x-1)}$

4. Evaluate the following integrals

(a) $\int_1^2 s^3\,ds$

(b) $\displaystyle\int \frac{2-3u}{\sqrt{u}}\,du$

(c) $\displaystyle\int \frac{1}{3x-7}\,dx$

(d) $\displaystyle\int_{-3}^{3} \frac{\sin(x)x^6}{1+x^4}\,dx$

(e) $\displaystyle\int (\sin x)^4(\cos x)^3\,dx$

(f) $\displaystyle\int x^2(1-x^3)^7\,dx$

(g) $\displaystyle\int t^{1/2}\ln(t)\,dt$

(h) $\displaystyle\int \frac{x^4}{x^2+1}\,dx$

(i) $\displaystyle\int \frac{1}{x^2-1}\,dx$

5. Find the derivative of the function

$$g(x) = \int_0^{1/x} \frac{3}{t^2+\ln(t+2)}\,dt$$

Bonus. Prove the following statement. If

$$\int_{-x}^{x} f(t)\,dt = 0 \qquad \text{for all } x > 0$$

then f is an odd function: $f(-x) = -f(x)$.