MA 126, Fall 2004

FINAL EXAM

December 14, 2004 (150 minutes)

Name: SSN:

Max. Points: 100 + 10 Bonus Points: Exam Grade:

Turn in **all the work** which you did to solve the problems, not just the final answer. In particular, include **intermediate steps in calculations**, wherever they demonstrate which method you used to get the result. You may use separate sheets for this.

The exam is **closed book** and **closed notes**. No calculator is to be used.

1. Evaluate the following definite, indefinite and improper integrals (5P+5P+5P+5P+5P):

(a) $\int e^x \sin(e^x)\,dx$

(b) $\int xe^{3x}\,dx$

(c) $\int \frac{1}{x(1+x)}\, dx$

(d) $\int_1^e \frac{(\ln x)^3}{x}\, dx$

(e) $\int_0^\infty \frac{x}{(1+x^2)^2}\, dx$

2. (a) Find the area enclosed between the two curves $y = x^2$ and $y = x$. (4P)

(b) A solid is generated by revolving the region between the curves $y = x^2$ and $y = x$ about the x-axis. Find its volume. (4P)

3. Find the limits of the following sequences (3P+3P+3P):

(a) $\lim_{n\to\infty} \frac{n^3}{1-2n^3}$

(b) $\lim_{n\to\infty} \frac{(-1)^n}{n}$

(c) $\lim_{n\to\infty} \frac{e^n}{1+e^n}$

4. Do the following series converge or diverge? Find the sum of those which converge (4P+4P).

(a) $\sum_{n=0}^{\infty} 3\left(\frac{1}{\sqrt{2}}\right)^n$

(b) $\sum_{n=1}^{\infty} \frac{1}{3}(\sqrt{2})^n$

5. Do the following series converge or diverge? Justify your answer by referring to the tests which were used (4P+4P+4P+4P).

(a) $\sum_{n=1}^{\infty} \frac{1}{n^2 + \ln n}$

(b) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n} + 3}$

(c) $\sum_{n=1}^{\infty} \frac{e^n}{e^n + 1000}$

(d) $\sum_{n=2}^{\infty} (-1)^n \frac{1}{\ln n}$

6. Find the radius and interval of convergence for the power series (6P)

$$\sum_{n=1}^{\infty} \frac{3^n x^n}{n^2}$$

7. Find the second order Taylor polynomial $T_2(x)$ for $f(x) = \sqrt{x}$ at $a = 1$. (4P)

8. Let $\mathbf{a} = \langle 1, -1, 1 \rangle$, $\mathbf{b} = \langle 3, 2, 1 \rangle$ and $\mathbf{c} = \langle 2, 2, 0 \rangle$. Find

(a) the cosine of the angle between $\mathbf{a}$ and $\mathbf{b}$ (3P),

(b) the vector projection of $\mathbf{b}$ onto $\mathbf{a}$ (3P),

(c) The area of the parallelogram determined by $\mathbf{a}$ and $\mathbf{b}$ (3P),

(d) The volume of the parallelepiped (skew box) determined by $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ (3P).

9. Find an equation for the plane which passes through the points $P(-1,-1,-1)$, $Q(1,1,1)$ and $R(0,2,3)$. (6P)

10. Find parametric equations for the line of intersection of the two planes $x-y+z=2$ and $x+y-2z=1$. (6P)

11*. Find a positive number c such that $\sum_{n=1}^{\infty}(1+c)^{-n} = 2$. (5P*)

12*. Use the Fundamental Theorem of Calculus to show the following: If the function f is continuous and if $\int_{-x}^{x} f(t)\,dt = 0$ for every $x > 0$, then f is odd (meaning that $f(-x) = -f(x)$ for every $x > 0$). (5P*)