MA 126, Fall 2004

TEST # 1

September 23, 2004 (105 minutes)

Name: SSN:

Max. Points: 100 + 8 Bonus Points: Test Grade:

Turn in **all the work** which you did to solve the problems, not just the final answer. In particular, include **intermediate steps in calculations**, wherever they demonstrate which method you used to get the result. You may use separate sheets for this or write on the back of a page.

The test is **closed book** and **closed notes**. **NO** calculator is to be used.

1. Find the following derivatives (5P+5P):

(a) $\dfrac{d}{dx}\displaystyle\int_1^x \sin(t^2)\,dt$

(b) $\dfrac{d}{dx}\displaystyle\int_0^{\sin x} \dfrac{dt}{1+t^4}$

2. Evaluate the following indefinite integrals (6P+6P+6P+6P):

(a) $\int 3x^2\sqrt{x^3+1}\,dx$

(b) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$

(c) $\int \frac{t}{t^2-t-2}\,dt$

(d) $\int \sin\sqrt{x}\,dx$

(Hint: Do a substitution followed by an integration by parts.)

3. Evaluate the following definite integrals (6P+6P+6P+6P):

(a) $\int_0^1 \frac{2x}{\sqrt{x^2+1}}\,dx$

(b) $\int_0^{\pi/8} \cos(4t)\,dt$

(c) $\int_0^1 x^2 e^x \, dx$

(d) $\int_{-1}^1 \sin(x^3) \, dx$

4. (a) Use the Trapezoidal Rule and the Midpoint Rule with $n = 2$ subintervals to find the approximate values T_2 and M_2 for the integral (4P+4P)

$$\int_1^2 x^2 \, dx.$$

(b) Explain geometrically and in words why T_2 is an overestimate for the exact value of $\int_1^2 x^2\,dx$ (without calculating the integral, which is easy). (4P)

(c) The error bound formula for the Midpoint Rule with n subintervals is

$$|E_{M_n}| \leq \frac{K(b-a)^3}{24n^2},$$

where it must hold that $K \geq |f''(x)|$ for all $a \leq x \leq b$.
For the integral from (a) find a bound for $|E_{M_5}|$. (4P)

(d) For the integral from (a) and with K found in (c), find the smallest possible n, such that it is guaranteed that $|E_{M_n}| < \frac{1}{1200}$. (4P)

4. Calculate the values of the following improper integrals or show that they diverge (7P+7P):

(a) $\int_0^1 \frac{dx}{x^2}$

(b) $\int_0^\infty xe^{-x^2}\,dx$

5. Is the following improper integral convergent or divergent? (8P)

$$\int_e^\infty \frac{\ln x}{x}\,dx$$

6.* If f is a continuous function such that

$$\int_0^x f(t)\,dt = xe^{2x} + \int_0^x e^{-t} f(t)\,dt$$

for all x, find an explicit expression for $f(x)$. (8P*)