Calculus I

# TEST 2A

October 20<sup>th</sup>, 2004

Name: _____

- Show your work; clearly write down each step in your calculation/reasoning. *No credit* is given for a correct numerical answer without any justification.

1. (a) *(4pts)* Differentiate
$$f(x) = (x^2 - 45)^3$$

(b) *(6pts)* Find the points of inflection of $f$.

2. *(10pts)* Differentiate
$$f(x) = \frac{e^{3x}}{1 + \ln x}$$

3. *(10pts)* Differentiate
$$y = \sin^2 (3x^2 - 2x)$$

4. *(10pts)* Differentiate
$$y = (\cos x)^x$$

(Hint: use logarithmic differentiation)

5. *(10pts)* Use logarithmic differentiation to differentiate the following:

$$y = \frac{e^x (2x + 1)\sqrt{x}}{x^2 - 1}$$

(You don't have to find a common denominator.)

6. *(5pts)* Suppose that $h(x) = f(g(x))$. Find $h'(1)$ if $f'(5) = 2$ and $g(x) = 4x^2 + x$.

7. *(10pts)* Find the point(s) of the curve $y = e^x(x^2 + 4x + 5)$ where the tangent line is horizontal.

8. *(10pts)* Find the equation of the tangent line at the point $(-5, \frac{9}{4})$ for the following curve:

$$\frac{x^2}{16} - \frac{y^2}{9} = 1$$

9. (a) *(7pts)* Use Newton's method with $x_1 = 1.2$ to show that $\sqrt[4]{2} \approx 1.18920712$.

(b) *(3pts)* Starting with $x_1 = 1.2$, at what $x_n$ can you stop, if you would like to know $\sqrt[4]{2}$ correct to 6 decimal places? Explain your answer.

10. The revenue $R(x)$ for a company when producing $x$ units is given by the equation

$$R(x) = xp(x)$$

where $p(x)$ is the selling price when the company produces $x$ units. The function $p(x)$ is assumed to be differentiable.

(a) *(5pts)* Suppose that $p(1000) = 10.50$, $p'(1000) = 0.002$. What is $R'(1000)$?

(b) *(5pts)* Use linear approximation to estimate $R(1005)$, the revenue when producing 1005 units.

(c) *(5pts)* Suppose the company is producing $x$ units. Use differentials to approximate the change in revenue ($\Delta R$) when the company will start to produce $x + 1$ items. Express your answer in terms of $p(x)$ and $p'(x)$ only.