Math 227 FINAL EXAM

Do not use any books or notes. You can use a calculator, but not graphing calculator. If you use a calculator, leave your results in exact form instead of decimal form. **Show all work for full credit.**

1. Find the velocity, acceleration, and speed of a particle with the given position function $\mathbf{r}(t) = \sqrt{2}t\,\mathbf{i} + e^t\,\mathbf{j} + e^{-t}\,\mathbf{k}$. (5 points)

2. Find the limit, if it exists, or show that the limit does not exist. You need to justify your answer. (15 points)

   (a) $\displaystyle\lim_{(x,y)\to(5,-2)} (x^5 + 4x^3y - 5xy^2)$

   (b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{8x^2y^2}{x^4 + y^4}$

   (c) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2 + y^2}{\sqrt{x^2 + y^2 + 1} - 1}$

3. Find the linear approximation of the function $f(x, y) = \sqrt{20 - x^2 - 7y^2}$ at $(2, 1)$ and use it to approximate $\sqrt{20 - 1.95^2 - 7(1.08)^2}$. (8 points)

4. Use the Chain Rule to find $\partial z/\partial s$ and $\partial z/\partial t$. (14 points)

(a) $z = x^2 + xy + y^2, \quad x = s + t, \quad y = st$ (b) $z = e^x \cos y, \quad x = st, \quad y = \sqrt{s^2 + t^2}$

5. The equation $xyz = \cos(x + y + z)$ defines z as a function of x and y. Find $\partial z/\partial x$ and $\partial z/\partial y$. (8 points)

6. Find the maximum rate of change and the direction in which it occurs for $f(x, y) = \ln(x^2+y^2)$ at the point $(1, 2)$. (8 poins)

7. (22 points)

(a) Evaluate $\iiint_E x^2 \, dV$, where E is the solid that lies within the cylinder $x^2 + y^2 = 1$, above the plane $z = 0$, and below the cone $z^2 = 4x^2 + 4y^2$.

(b) Use spherical coordinates to evaluate

$$\int_0^2 \int_0^{\sqrt{4-x^2}} \int_0^{\sqrt{4-x^2-y^2}} (x^2 + y^2 + z^2) \, dzdydx$$

8. (12 points)

(a) Determine whether or not $\mathbf{F}(x, y) = (2x\cos y - y\cos x)\,\mathbf{i} + (-x^2\sin y - \sin x)\,\mathbf{j}$ is a conservative vector field.

(b) If it is, find a function f such that $\mathbf{F} = \nabla f$.

(c) Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ along curve C: C is the upper semicircle that starts at $(0, 1)$ and ends at $(2, 1)$. Can you use your results in (a) and/or (b)?

9. Use Green's Theorem to evaluate $\int_C xy\,dx + 2x^2\,dy$, where C is positively oriented and consists of the line segment from $(-2, 0)$ to $(2, 0)$ and the top half of the circle $x^2 + y^2 = 4$. (8 points)

10. **Bonus** (10 points extra) Apply the second vector form of Green's Theorem to $\mathbf{F}(x, y) = x\,\mathbf{i} + y\,\mathbf{j}$ and C given by $x^2 + y^2 = 4$, and express A in terms of s, where A is the area of the region bounded by C and s is the circumference of C.