MA 227: Calculus III
Final Test, December 11, 2003

Timing: 10:45—1:15

Your name:

Your signature:

1. Let $u = xy + yz + zx$, $x = st$, $y = e^{st}$, $z = t^2$. Calculate $\partial u/\partial s$ and $\partial u/\partial t$ when $s = 1$, $t = 1$.

10 points

2. Find the maximum rate of change of the function $f(x, y, z) = x^2y^3z^4$ at the point $(-1, -1, -1)$ and the direction in which it occurs. (The direction should be characterized by the unit vector having that direction.)

10 points

3. Find the absolute maximum and minimum values of $f(x, y) = xy^2$ on the unit disk of radius 2 centered at the origin.

10 points

4. Find the maximum and minimum values of $f(x, y, z) = 3x - y - 3z$ subject to the constraints $x + y - z = 0$ and $x^2 + 2z^2 = 1$.

10 points

5. Find the volume of the solid bounded by the elliptic paraboloid $z = (x-1)^2 + 4y^2$, the planes $x = 3$ and $y = 3$, and the coordinate planes.

10 points

6. Find the double integral of the function $f(x, y) = ye^x$ on the triangular region with vertices $(0, 0)$, $(3, 0)$, and $(1, 1)$.

10 points

7. Find the volume of the solid that is inside the cylinder $x^2 + y^2 = 4$ and the ellipsoid $4x^2 + 4y^2 + \frac{z^2}{4} = 64$.

10 points

8. A lamina is defined by the inequalities $x^2 + y^2 \leq 4$, $x \geq 0$, $y \geq 0$. The mass density is $\rho(x, y) = x^2 + y^2$. Find the moments of inertia I_x, I_y, I_0.

10 points

9. Evaluate

$$\iiint_E (x + 3y)dV,$$

where E is bounded by the parabolic cylinder $y = x^2$, and the planes $x = z$, $x = y$, $z = 0$.

10 points

10. Evaluate

$$\iiint_E xe^{(x^2+y^2+z^2)^2} dV,$$

where E is the solid that lies between the spheres $x^2+y^2+z^2 = 1$ and $x^2+y^2+z^2 = 9$ in the first octant.

10 points