

MA 227, Calculus - III.

Midterm test - I

Monday, October 6, 2003.

Student's Name _____

(Please, print)

GIVE REASONS FOR YOUR ANSWERS!

TEST 1:

HW:

The Final Grade for TEST 1:

I. (15%) A helix is described by the equation

$$\vec{r}(t) = (4 \sin t, 4 \cos t, 3t).$$

- a) Find $\vec{r}'(t)$ and $\vec{r}''(t)$.
- b) Find the length of the curve when $0 \leq t \leq \pi$.
- c) Find the curvature at the point $t = 0$.

II. (15%) A helix is described by the equation

$$\vec{r}(t) = (4 \sin t, 4 \cos t, 3t).$$

- a) Find the tangent vector \vec{T} at the point $t = 0$.
- b) Find the normal vector \vec{N} at the point $t = 0$.
- c) Find the binormal vector \vec{B} at the point $t = 0$.

III. (15%) A helix is described by the equation

$$\vec{r}(t) = (4 \sin t, 4 \cos t, 3t).$$

- a) Find the equation of the normal plane at $t = 0$.
- b) Find the equation of the osculating plane at the point $t = 0$.
- c) Find the angle between the helix and the line $\vec{r}(s) = (s, s + 4, -s)$ at the point $(0, 4, 0)$.

IV. (15 %) The motion of the particle is described by the equation

$$\vec{r}(t) = (4 \sin t, 4 \cos t, 3t).$$

- a) Find the velocity and the acceleration as functions of time.
- b) Find the speed at $t = 0$.
- c) Find the tangential and normal component of acceleration at $t = 0$.

V. (10%) Find the integral $\int_0^1 (5\vec{i} - 2t\vec{j} + t^2\vec{k}) dt$.

VI. (10%) Let $f(x, y) = x^2 + 3yx^3$. Find f_x , f_y , f_{xx} , f_{yy} , f_{xy} .

VII (20%). Let $f(x, y) = x^2 + y^2$.

- a) Find the equation of the tangent plane to the surface $z = x^2 + y^2$ at the point $(2, 1)$.
- b) Find the linearization of the function $f(x, y) = x^2 + y^2$ at the point $(2, 1)$.
- c) Use the linearization to find an approximate value of the function at the point $(2.2, 1.3)$.
- d) Find the formula for dz at the point $(2, 1)$.