Name: ______________________

Calculus 2

MA126-6B

Final Examination

Thursday, December 11, 2003

Instruction: Answer the questions in the space provided. Use the scratch paper provided if needed. Please keep your answers neat, complete but brief, and to the point.

Question 1	________
Question 2	________
Question 3	________
Question 4	________
Question 5	________
Question 6	________
Question 7	________
Question 8	________
Question 9	________
Question 10	________
Question 11	________
Question 12	________
Question 13	________
Question 14	________
Question 15	________
Total	________

Please do not write in this box

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QUESTION 1. Evaluate the integral:

$$\int_0^{\pi/4} \frac{x}{\cos^2 x}\, dx.$$

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QUESTION 2. Evaluate the integral:

$$\int \frac{dx}{x(x^2+1)}.$$

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QUESTION 3. The midpoint method M_n is used to approximate the following integral:

$$\int_0^1 e^{x^3}\,dx.$$

How large should one choose n in order to guarantee the error is less than 10^{-6}?
Hint: Recall that the error in the midpoint method can be estimated by:

$$|E_M| \leq \frac{K(b-a)^3}{24n^2}.$$

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QUESTION 4. Determine whether the following improper integral converges:

$$\int_0^1 \frac{\sqrt{x^2+1}}{x}\, dx.$$

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QUESTION 5. Find the area bounded between the two curves:

$$y = \sqrt{x}, \qquad y = |x - 2|.$$

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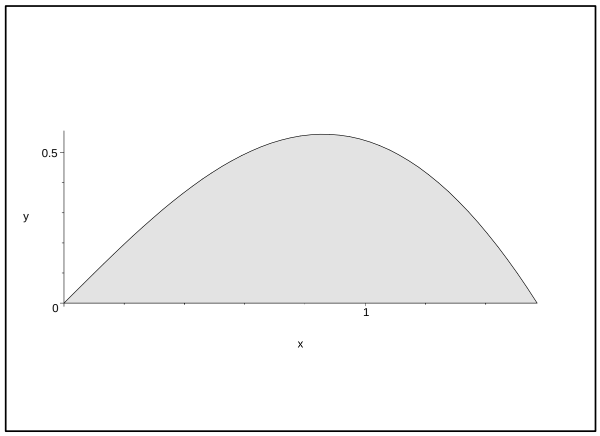
QUESTION 6. Find the volume of the solid of revolution obtained by rotating the area under the curve

$$y = x\cos x, \qquad 0 \le x \le \pi/2,$$

about the y-axis:



Hint: Use cylindrical shells.

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QUESTION 7. Find the arclength of the curve:

$$x = y^{3/2}, \qquad 0 \leq y \leq 1.$$

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QUESTION 8. Check that the function:

$$f(x) = \begin{cases} \frac{1}{2}\sin x & \text{if } 0 \leq x \leq \pi \\ 0 & \text{otherwise} \end{cases}$$

is a probability density function. Find the mean, standard deviation, and median.

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QUESTION 9. Determine whether the sequence $\left\{\left(1+\frac{3}{n}\right)^{4n}\right\}_{n=1}^{\infty}$ converges, and if it does, find its limit. Justify your answer.

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QUESTION 10. Determine whether the following series converges, and if it does, find its sum:

$$\sum_{n=2}^{\infty} \left(-\frac{2}{3}\right)^n$$

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QUESTION 11. Determine whether the following series converges:

$$\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right).$$

Justify your answer.

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QUESTION 12. Determine whether the following series converges, converges absolutely, or converges conditionally:

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}.$$

Hint: Use the integral test.

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QUESTION 13. Find the Maclaurin series for the function:

$$f = \frac{1}{(1-x)^2}.$$

Determine the interval of convergence.
Hint: $1/(1-x)^2$ *is the derivative of* $1/(1-x)$.

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QUESTION 14. Find the Maclaurin series for the function:

$$f(x) = \ln(1 - x).$$

Determine its interval of convergence.
Hint: $\ln(1 - x)$ is the indefinite integral of $-1/(1 - x)$.

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QUESTION 15. Check that the series

$$\sum_{n=0}^{\infty} \frac{1}{(2n)!}$$

converges, and find its sum.
Hint: Find the Maclaurin series of $\cosh x = (e^x + x^{-x})/2$.