Calculus 2

MA126-6B

Midterm Examination 2

Tuesday, November 18, 2003

Instruction: Answer the questions in the space provided. Use the scratch paper provided if needed. Please keep your answers neat, complete but brief, and to the point.

QUESTION 1. Find the volume of the solid of revolution obtained by rotating the area under the curve

 $y = x \cos x, \qquad 0 \le x \le \pi/2,$

about the y-axis:



Hint: Use cylindrical shells.

QUESTION 2. Find the area bounded by the two curves:

$$y^2 = x + 2, \qquad y = |x|.$$

QUESTION 3. Find the arclength of the curve:

$$x = y^{3/2}, \qquad 0 \le y \le 1.$$

QUESTION 4. Check that the function:

$$f(x) = \begin{cases} 6x(1-x) & \text{if } 0 \le x \le 1\\ 0 & \text{otherwise} \end{cases}$$

is a probability density function. Find the mean, standard deviation, and median.

QUESTION 5. Find the limit:

$$\lim_{n \to \infty} \frac{n \cos n}{n^2 + 1}.$$

Justify you answer.

QUESTION 6. Determine whether the following series converges:

$$\sum_{n=1}^{\infty} \ln\left(1 + \frac{1}{n}\right).$$

Justify your answer.

QUESTION 7. Determine whether the following series converges, converges absolutely, or converges conditionally:

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}.$$

Hint: Use the integral test.

QUESTION 8. Find the Maclaurin series for the function:

$$f = \frac{1}{(1-x)^2}.$$

Determine the interval of convergence.

Hint: $1/(1-x)^2$ is the derivative of 1/(1-x).

QUESTION 9. Let $f(x) = x \arctan x$. Find $f^{(126)}(0)$, the 126th derivative of f at x = 0. Hint: Find the Maclaurin series of f, and use Taylor's formula for the 126th coefficient.

QUESTION 10. Check that the series

$$\sum_{n=0}^{\infty} \frac{1}{(2n)!}$$

converges, and find its sum.

Hint: Find the Maclaurin series of $\cosh x = (e^x + x^{-x})/2$.