Mathematics 126 **Midterm 3**
Nov. 14, 2003

- Calculators are allowed *only* for numerical calculations.

- There are two sheets of scratch paper attached at the end of the exam. Use them and but do not tear them off the exam in doing so, and hand them in together with the exam.

- Show your work; clearly write down each step in your calculations/reasonings.

1. The parametrized curve $x = e^{-t} \sin t, y = e^{-t} \cos t,\ 0 \leq t \leq \infty$ on the xy-plane gives a curve spiralling into the origin as $t \to \infty$.
a) Let l_n be the length of the portion of the curve while t changes from $(n-1)\pi$ to $n\pi$. Write down and then evaluate an integral representing the length l_n.

b) Show that the total length of the spiral $\sum_{n=1}^{\infty} l_n$ is a geometric series. Is it convergent?

2. The temperature at t AM is given by the function

$$T(t) = 50 - 14 \sin \frac{\pi t}{12}.$$

(For example $T(1)$ would be the temperature at 1AM.) Find the average temperature from midnight to 6AM.

3. Determine whether the following *sequence* converges or not. If it converges, find its limit. Explain your reasoning.

a)

$$a_n = \arctan n^2$$

b)

$$a_n = \frac{1}{n} \cos n\pi$$

4. Determine whether each *series* converges or not. Explain your reason.

a)

$$\sum_{n=1}^{\infty}\left[\sin\left(\frac{1}{n}\right)-\sin\left(\frac{1}{n+1}\right)\right]$$

b)

$$\sum_{n=1}^{\infty}\frac{2^n}{n^2}$$

c)

$$1 + \frac{\ln 2}{2} + \frac{\ln 3}{3} + \frac{\ln 4}{4} + \frac{\ln 5}{5} + ...$$

d)

$$1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \frac{1}{\sqrt{5}} - ...$$

5. Use the integral test to determine if the series $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$. You need to explain each step explicitly. In particular, draw the graph of the step function you would be using in the xy plane given below.
Hint: Consider the graph of $y = \frac{1}{x(\ln x)^2}$.